EXAMINATION II:

Fixed Income Valuation and Analysis

Derivatives Valuation and Analysis

Portfolio Management

Solutions

Final Examination

March 2014

Question 1: Fixed Income Valuation and Analysis

a)

a1)

$$P = \frac{100}{(1+y)^T} = \frac{100}{(1+1.75\%)^{30}} = 59.42$$

$$D^{mod} = \frac{D}{1+y} = \frac{30}{1+1.75\%} = 29.48$$

$$C = \frac{1}{P} \cdot \frac{1}{(1+y)^2} \sum_{t=1}^{T} t \cdot (t+1) \cdot \frac{CF_t}{(1+y)^t} \overset{\text{for a zero bond}}{=} \frac{1}{P} \cdot \frac{1}{(1+y)^2} T \cdot (T+1) \cdot \frac{100}{(1+y)^T} = \frac{T \cdot (T+1)}{(1+y)^2}$$

$$\Rightarrow C = \frac{30 \cdot 31}{(1+1.75\%)^2} = 898.28$$

a2)

The absolute price change ΔP of the bond in case its yield increased by Δy = 125 bps by taking into account both its modified duration D^{mod} and convexity C is [we assume a notional amount of 100]:

$$\Delta P \approx P \cdot \left(-D^{mod} \cdot \Delta y + \frac{1}{2} \cdot C \cdot \Delta y^2 \right) = 59.42 \cdot \left(-29.48 \cdot 0.0125 + \frac{1}{2} \cdot 898.28 \cdot 0.0125^2 \right) = -17.73$$

a3)

Approx. new price based only on D^{mod}:

$$P^{new} \approx P \cdot (1 - D^{mod} \cdot \Delta y) = 59.42 \cdot (1 - 29.48 \cdot 0.0125) = 37.52$$

Exact new price [new yield y^{new} = 1.75% + 1.25% = 3%]:

$$P = \frac{100}{(1+3\%)^{30}} = 41.20$$

Duration + convexity = 59.42 - 17.73 = 41.69

The result when approximating via duration and convexity is fairly reliable as higher differentials/moments have very small impacts.

b)

b1)

Let w_1 be the weight of the 4% Corporate Bond, w_2 the weight of the 2% Covered Bond and w_3 the weight of the 0% Government Bond. Then:

$$\begin{cases} w_3 = 1 - w_1 - w_2 & (1) \\ w_1 \cdot 6.04 + w_2 \cdot 8.92 + (1 - w_1 - w_2) \cdot 29.48 = 12 & (2) \\ w_1 \cdot 36 + w_2 \cdot 73 + (1 - w_1 - w_2) \cdot 898 = 200 & (3) \end{cases}$$

$$\begin{cases} w_1 \cdot (6.04 - 29.48) + w_2 \cdot (8.92 - 29.48) = 12 - 29.48 \\ w_1 \cdot (36 - 898) + w_2 \cdot (73 - 898) = 200 - 898 \end{cases} \Rightarrow \begin{cases} w_1 \cdot 23.44 + w_2 \cdot 20.56 = 17.48 & (2') \\ w_1 \cdot 862 + w_2 \cdot 825 = 698 & (3') \end{cases}$$

$$\Rightarrow \begin{cases} w_1 = \dfrac{17.48 - w_2 \cdot 20.56}{23.44} & (2') \\ \left(\dfrac{17.48 - w_2 \cdot 20.56}{23.44}\right) \cdot 862 + w_2 \cdot 825 = 698 & (3') \end{cases}$$

$$\Rightarrow w_2 \cdot \left(825 - \frac{20.56}{23.44} \cdot 862\right) = 698 - \frac{17.48}{23.44} \cdot 862 \Rightarrow w_2 = \frac{698 - \frac{17.48}{23.44} \cdot 862}{\left(825 - \frac{20.56}{23.44} \cdot 862\right)} = 0.8001 = 80.1\%$$

$$\Rightarrow w_1 = \frac{17.48 - 0.8001 \cdot 20.56}{23.44} = 4.4\% \Rightarrow w_3 = 100\% - 80.1\% - 4.4\% = 15.5\%$$

b2)

Practical problem: With 3 bonds (hence 3 unknowns w_1, w_2 and w_3) and 2 constraints (sum of weights =1, and portfolio mod. duration = 12) there are infinite solutions (i.e. no clearly defined answer)

Hedge problem: Hedging pension assets/liabilities solely based on duration only works for small parallel interest rate movements (i.e. bad hedge results for larger and/or non-parallel movements) and with a flat term structure.

c)

c1)

- Credit risk [spread risk]: The pension assets portfolio includes a corporate bond which is possibly different from the bond universe that constitutes the pension liabilities.
- Basis risk: Your pension assets include covered and government bonds, which are not part of the bond universe that comprises the pension liabilities.
- Default risk: The pension assets portfolio is subject to default whereas possibly a bond that is currently part of the bond universe that comprises your pension liabilities is discarded when its credit quality goes below a given threshold, hence before default.
- Inflation risk: The pension assets could lose in value with rising inflation rates, whereas the pension liability benchmark could contain inflation linked bonds.
- Liquidity risk: low/no turnover of the pension assets bonds in the secondary markets could cause very large bid-ask spreads and therefore liquidity risk.
- …..

c2)

- Credit risk [spread risk] => Hedge: Do not only invest in one corporate bond but diversify your investments across corporate bonds that comprise your pension liabilities!
- Basis risk => Hedge: Only invest in corporate bonds that comprise your pension liabilities!
- Default risk => Hedge: Buy credit protection (CDS) pertaining to the critical bonds in your pension fund!
- Inflation risk => Hedge: Buy inflation-linked bonds or (as there are hardly any inflation protected corporate bonds) enter into inflation swaps.
- Liquidity risk: buy only liquid securities with a large issue volume.
- ..

Question 2: Fixed Income/Derivatives Valuation and Analysis

a)

a1)

The real price is: $P^{real} = \frac{2}{1+1\%} + \frac{2}{(1+1\%)^2} + \frac{102}{(1+1\%)^3} = 102.94$.

Multiplying it with the CPI index ratio of 1.10 (= 110 / 100), we obtain the actual purchase price (nominal): 102.94 · 1.1 = 113.24.

a2)

Real yield at both the time of purchase and sale was 1.0%, therefore the real holding period return for the 1-year period is 1.0%. The nominal holding period return is found by multiplying the real holding period return by the CPI growth rate: $1.01 \cdot 1.015 - 1 = 2.5\%$.

Alternative solution:

The real price after one year is: $P^{real} = \frac{2}{1+1\%} + \frac{102}{(1+1\%)^2} = 101.97$.

Multiplying it with the CPI index ratio of 1.10 · 1.015 (= 1.1165), we obtain the actual sell price (nominal): 101.97 · 1.1165 = 113.85. The coupon which is earned is: 2% · 111.65 = 2.233. Therefore the HPR is: $HPR = \frac{2.233 + 113.85}{113.24} - 1 = 2.5\%$.

b)

b1)

$$1 + R^{real} = \frac{1 + R^{nom}}{1 + \pi} \Rightarrow \pi = \frac{1 + R^{nom}}{1 + R^{real}} - 1 = \frac{1 + 2\%}{1 + 1\%} - 1 = 1.0\%$$

b2)

If the rate of inflation π will be greater than 0.99%, the return on the inflation-indexed bond will be higher than the return on the government bond, since: $1 + R^{nom} = \left(1 + R^{real}\right) \cdot (1 + \pi)$.

b3)

Liquidity: Inflation-indexed government bonds generally lack liquidity; this lowers their price and therefore increases their yield.

Generally, inflation rate risk will be lower for index linked bonds relative to conventional bonds due to the linking of interest and redemption values to inflation measures, although the linking occurs with a slight lag. However, there is no guarantee that the CPI growth rate will perform as expected, and because of this risk of fluctuation, both the prices of inflation-indexed and conventional government bonds will be correspondingly lower, and their yields higher than if the CPI were not subject to such fluctuations or differences between expectations and realizations.

c)

c1)

US-style inflation-indexed government bonds come with a floor for the investor, which is essentially a put option, therefore their price is higher.

c2)
Inflation rate: The lower the CPI growth rate (especially at levels near zero, or as negatives grow larger), the higher the value of the put option and the higher the value of US-style bonds.

CPI fluctuation risk: The higher the CPI fluctuation risk, the higher the value of the option, and the higher the value of US-style bonds.

c3)
With US-style bonds, when the CPI growth rate moves from 1% to 2-3%, the put option becomes increasingly out-of-the-money, and its value declines, therefore it offsets a portion of the rise in nominal bond prices caused by rising consumer prices. So the UK-style bonds will produce higher returns.

Question 3: Derivatives Valuation and Analysis

a)

$$C_E - P_E - S + Ke^{-r\tau} \overset{!}{=} 0$$

$$K = 70: \quad 13.17 - 2.47 - 80 + 70 \cdot e^{-1\% \cdot 1} = -69.30 + 70 \cdot 0.99 = 0$$

$$K = 80: \quad 7.31 - 6.51 - 80 + 80 \cdot e^{-1\% \cdot 1} = -79.20 + 80 \cdot 0.99 = 0$$

$$K = 90: \quad 3.66 - 14.76 - 80 + 90 \cdot e^{-1\% \cdot 1} = -91.10 + 90 \cdot 0.99 = -2 \neq 0$$

The options with K = 90 violate the put-call parity.

b)
b1) + b2)
The put with strike K = 90 is overvalued with respect to the call [since $P_K > C_K - S + Ke^{-rT}$] Therefore you buy the synthetic put [i.e., from put-call parity $P_K = C_K - S + Ke^{-rT}$, you buy the call, short the underlying stock, invest the present value of the strike] and sell the real one. You trade 10 contracts with a contract size of 10 EUR each [i.e. you work on 100 underlyings]:

b1) Transactions	b1) Today (t = 0)	b2) At maturity (t = T)	
		$S_T \geq 90$	$S_T < 90$
1. Buy 10 call contracts with K = 90	– 366	$100 \cdot (S_T - 90)$	0
2. Sell 10 put contracts with K = 90	+ 1,476	0	$-100 \cdot (90 - S_T)$
3. Sell short 100 stocks at S = 80	+ 8,000	$-100 \cdot S_T$	$-100 \cdot S_T$
4. Invest 100 times the PV of K = 90 [$90 \cdot e^{-1\%} = 89.1$]	– 8,910	100·90	100·90
Total:	200	0	0

Today you realize an immediate arbitrage profit of 200 EUR.

c)
c1)
The initial cost of the strategy is:

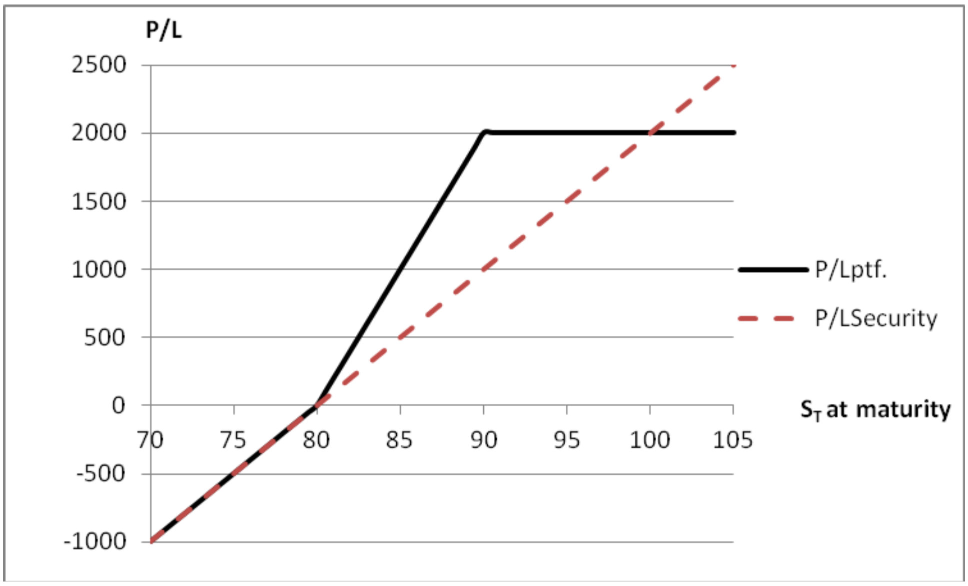
$$100 \cdot [80 + 7.31 - 2 \cdot 3.66] = 100 \cdot [79.99] = 7{,}999$$

The profit/loss at expiration is:

$$P\,\&\,L = \begin{cases} 100 \cdot (S_T - 79.99) & (S_T < 80) \\ 100 \cdot (S_T - 79.99) + 100 \cdot (S_T - 80) & (80 \leq S_T \leq 90) \\ 100 \cdot (10.01) + 100 \cdot (10) = 100 \cdot 20.01 = 2{,}001 & (S_T > 90) \end{cases}$$

The maximum loss is: $100 \cdot 79.99 = 7{,}999$ (when $S_T = 0$).
The maximum profit is 2’001 (when $S_T \geq 90$).
The upward break-even point is at 79.99.

c2)



d)
This strategy is suitable for an investor expecting the underlying to rise moderately up to the cap. In fact the strategy allows a doubling of the performance of the underlying up to the cap.

When below the strike, this strategy reflects the underlying price moves in a one to one fashion. Therefore its risk is comparable to a direct investment in the underlying. Any payouts attributable to the underlying are used in favor of the strategy.

e)
$$\Delta_{\text{Strat}} = 100 \cdot [1 + 0.56 - 2 \cdot 0.35] = 100 \cdot 0.86 = 86$$
$$\Gamma_{\text{Strat}} = 100 \cdot [0.023 - 2 \cdot 0.021] = 100 \cdot [-0.019] = -1.9$$
$$\Theta_{\text{Strat}} = 100 \cdot [-3.81 - 2 \cdot (-3.469)] = 100 \cdot [3.128] = 312.8$$
$$\nu_{\text{Strat}} = 100 \cdot [31.54 - 2 \cdot 29.63] = 100 \cdot [-27.72] = -2{,}772$$

f)
The current delta of the strategy is 86, smaller than the delta of a portfolio long 100 stocks [whose delta is 100]. Therefore in the short run the colleague is right.

The delta of the strategy indicates that if the price of the underlying stock rises by one unit the value of the portfolio changes approximately by EUR 86.

Gamma is negative. Hence, if the stock price increases the delta decreases and the "underperformance" becomes even worse.

Theta is positive. Therefore, the strategy gains value with the passing of time.

Vega is negative. If the volatility increases by 20%, the strategy value decreases by $\nu_{\text{Strat}} \cdot 20\% = -2{,}772 \cdot 20\% = -554.4$

Question 4: Portfolio Management

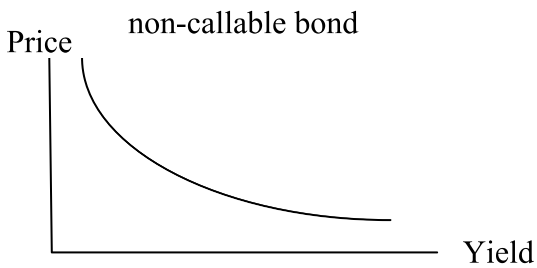
a)
i) Decreasing the maturity of your portfolio is **not** correct, since by doing this you also decrease its duration, and therefore you profit in a reduced way to the increase of the portfolio value as a consequence of the decrease in yields.

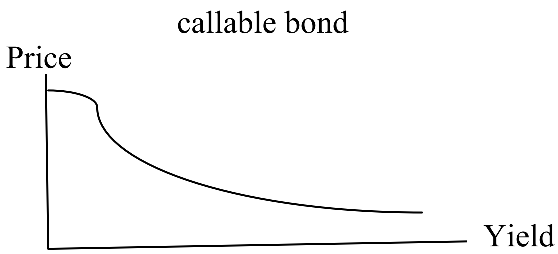
ii) Increasing yield to maturity of the bonds in the portfolio is indifferent/ **not** correct, since by increasing the YTM of the bonds you reduce the duration [and you possibly increase the credit risk of your bond portfolio]

iii) Increasing the duration of the portfolio **is** correct, since you profit in a larger way from a decrease of the yields.

b)
The bond which will profit (in absolute terms) the most from a decrease of the interest rates is bond ii) with Maturity 30 year, 9% coupon, 8% yield because it has the largest duration. This is because it has the largest maturity, lower coupon and lower yield, which all contribute to increase its duration.

c)





In a callable bond the call price establishes an upper bound for the price; therefore the price of a callable bond will not go much higher than this level. At high yields (i.e. low prices), the call is out of the money and this feature becomes less visible; but as yields decrease and the bond price approaches the call price, the probability of a call exercise will increase, and therefore the price is dampened.

d)
The relationship between yield and bond prices is inverse [when yields increase, bond prices decline]. The relationship between yield and the rate of return on reinvestment of interests is positive [when yields increase, interest received is reinvested at a higher yield].

	Bond prices	Reinvestment proceeds
Yields increase	decrease	increase
Yields decrease	increase	decrease

<u>Bond prices and reinvestment rates moves are in opposite directions; they will offset each other at a point in time</u>: Macauley **duration**. Duration is the point at which the drop (resp. increase) in price due to a rise (resp. decrease) in yield is offset by the higher (resp. lower) return from reinvestment of interests.

If the duration of a portfolio is set (and maintained) equal to the investment horizon, the two risks will offset and the value of the portfolio at the investment horizon will not be affected (as a first approximation, and assuming parallel movements of the yield curve) by changes in interest rates: the portfolio is **immunized**.

e)

e1)

Portfolio Sharpe ratio $S_P = \dfrac{R_P - R_f}{\sigma_P} = \dfrac{18\%}{20\%} = 0.9$

Market Sharpe ratio $S_M = \dfrac{R_M - R_f}{\sigma_M} = \dfrac{13\%}{12\%} = 1.0833$

The Sharpe ratio measures the reward per unit of risk, where risk is measured as total risk by the standard deviation (i.e., both diversifiable (unsystematic) and non-diversifiable (systematic) risk). Since the Sharpe ratio of the market is higher, this indicates that the reward (excess return) per unit of risk to the portfolio is less than that of the market, indicating that the portfolio has underperformed the market.

e2)

Portfolio Treynor ratio $T_P = \dfrac{R_P - R_f}{\beta_P} = \dfrac{18\%}{1.5} = 12\%$

Market Treynor ratio $T_M = \dfrac{R_M - R_f}{\beta_M} = \dfrac{13\%}{1} = 13\%$

The Treynor ratio measures the reward return per unit of risk, where risk is measured by the beta as non-diversifiable risk. Since the Treynor ratio of the market is higher, this indicates that the market has outperformed the portfolio.

e3)

The Jensen's alpha is the intercept of the regression, or $\alpha = -0.06$.

The regression is based on the CAPM relationship: ($R_p = R_f + \beta_p \cdot [R_M - R_f]$), where risk is defined as non-diversifiable risk. Since when performance is in line with the CAPM there is a zero intercept, the portfolio has underperformed the market.

e4)

Portfolio A is the sole holding of Mr. Adam, so there is no further diversification. Moreover, the portfolio holds a relatively small number of assets and is likely imperfectly diversified. Mr. Adam will experience the total risk, both diversifiable and non diversifiable, of the portfolio. In this situation, total risk is the applicable concept and the Sharpe ratio is the relevant measure. The Treynor ratio and Jensen's alpha are both based on non-diversifiable risk, which does not reflect the true risk experienced by the owner and would be less suitable in this situation.

Portfolio B is managed under a specific (growth) strategy. Although management under such a specific strategy may lead to imperfect diversification, the portfolio is not the owner's total holding and further diversification will be provided by the other 19 portfolios managed under different strategies. In this case the relevant risk concept is non-diversifiable risk. The Sharpe ratio, based on total risk, would be less suitable in this situation, while the Treynor ratio and Jensen's alpha, both based on non-diversifiable risk, would be more relevant.

f)

f1)

Stock prices are said to "follow a random walk" if the price changes can be described as a random variable from a probability distribution, such that the stock price change is independent of prior stock price changes.

f2)

A market is said to be efficient with respect to some set of information if that information is "fully reflected" in stock prices – i.e., the information cannot be used to consistently outperform the market index after considering the cost of information gathering and analysis.

f3)

In an efficient market, existing information is already reflected in the stock price. Stock price changes will be the result of the arrival of new information which, by definition, cannot be foreseen (if it could be foreseen, it would already be reflected in the price and would have no effect). If new information arrives randomly, the stock price changes caused by the new information would also be random.

f4)

The EMH defines three forms of efficiency. The forms are differentiated with respect to the information set to which the market is said to be efficient or to "fully reflect." These forms are:

- The weak form, in which the market is said to be efficient with respect to historical data.
- The semi-strong form, in which the market is said to be efficient with respect to publicly available data.
- The strong form, in which the market is said to be efficient with respect to all data, both public and private.

With regard to the specific sets of data:

i) Articles in the financial press are publicly available data and would fall under the **semi-strong** form of the EMH.

ii) Unannounced merger plans are not publicly available and would fall under the **strong** form of the EMH.

iii) Point and figure charts employ historical trading data and would fall under the **weak** form of the EMH.

iv) Financial ratio analysis employs the financial statements of the firm, which are publicly available data and would fall under the **semi-strong** form of the EMH.

Question 5: Portfolio Management

a)
Risk premium explanation:
Value stocks have lower P/E and P/B reflecting relatively poor financial conditions and earnings instability of underlying companies. Such lower valuation is due to a higher risk premium (as part of discount rate in DCF model) applied by investors.

Behavioral bias explanation:
Investors tend to overreact to news; overly optimistic to growth stories while overly pessimistic to bad news of earnings. This behavioral bias might make growth stocks priced much higher than their fundamentals justify, while it might make value stocks much cheaper than their intrinsic value.

b)
Betas are 1.03 for growth and 0.96 for value. If USD 100 million is held as a long-position in the value index, then let X be the short-position required for growth index in order to neutralize beta:

$$100 \cdot 0.96 - X \cdot 1.03 = 0 \Rightarrow X = 100 \cdot \frac{0.96}{1.03} = 93.2 \text{ Mio USD.}$$

The short position is USD 93.2 million.

c)
Total risk can be decomposed as follows: $\sigma_P{}^2 = \underbrace{\beta_P{}^2 \cdot \sigma_M{}^2}_{(\text{systematic risk})^2} + \underbrace{\sigma_\varepsilon{}^2}_{(\text{specific risk})^2}$.

The value index's beta is $\beta_P = 0.96$, total risk is $\sigma_P = 15\%$, and the market risk is $\sigma_M = 15\%$.
Therefore:
Sytematic risk = $\beta_P \cdot \sigma_M = 0.96 \cdot 15\% = 14.4\%$
Specific risk [style risk]: =

$$\sigma_\varepsilon = \sqrt{\sigma_P{}^2 - \beta_P{}^2 \cdot \sigma_M{}^2} = \sqrt{(15\%)^2 - (0.96 \cdot 15\%)^2} = 15\%\sqrt{1 - 0.96^2} = 4.2\%$$

d)
The monthly alpha of this market neutral strategy is 0.24 with a t-statistic of 2.75, which is statistically significant at the 95% confidence level. Therefore, this strategy seems to be valid (at least historically) at generating positive returns without market risk.

[Not required, only for pedagogical purposes:
Explanation about the t-statistic and the t-Student distribution:
Be X a random variable with mean μ_X and variance $\sigma_X{}^2$. Because of the Central Limit Theorem the sample average $\overline{X} = \frac{X_1 + X_2 + ... + X_n}{n}$ obtained from a random sample of n observations X_1, X_2, …,X_n of X follows a normal distribution: $\overline{X} \sim N(\mu_X, \frac{\sigma_X^2}{n})$. This means that $Z = \frac{\overline{X} - \mu_X}{\sigma_X / \sqrt{n}} \sim N(0,1)$ follows a standard normal.

But usually σ_X^2 is not known and must be estimated by the sample variance:

$$s_X^2 = \frac{\sum (X_i - \overline{X})^2}{n-1}.$$

So if we use the sample variance instead of the population variance, then:

$\frac{\overline{X} - \mu_X}{s_X / \sqrt{n}} \sim t_{n-1}$ follows a t-Student distribution with n-1 degrees of freedom.

The critical values of Student's t distribution with df degrees of freedom for the $\alpha = 95\%$, resp. $\alpha = 99\%$ confidence level are:

df	1	2	3	5	10	20	50	100	infinite
$\mathbf{t_{0.95,df}}$	6.314	2.92	2.353	2.015	1.812	1.725	1.676	1.66	1.645
$\mathbf{t_{0.99,df}}$	31.821	6.965	4.541	3.365	2.764	2.528	2.403	2.364	2.326

Here $X = \alpha = (R_P - R_f) - \beta \cdot (R_M - R_f)$.

If alpha would be distributed around zero [Null hypothesis], with df = 20 [i.e. when we consider 21 data in the regression calculation, so that n-1 = 20] and a confidence level of 99% we would observe a sample alpha lower than $\mathbf{t_{0.99,20}} = 2.528$; since the observed alpha is 2.75, we reject the null hypothesis and accept the alternative hypothesis that alpha > 0.

e)

In periods of crisis.

Since the market neutral strategy is not affected by the general market direction, it is particularly valuable under global market crash scenarios such as the Lehman crisis in 2008. In that year, both growth and value indexes had large negative returns but the market neutral strategy ended with a slightly positive return.

f)

In 1998 and 1999, the tech-bubble (dot.com boom) generated the higher return for growth stocks while value stocks were lagging behind. In these types of markets with investors' speculative expectations for higher future growth, the value style significantly underperforms growth style.