EXAMINATION II:

Fixed Income Valuation and Analysis

Derivatives Valuation and Analysis

Portfolio Management

Solutions

Final Examination

March 2016

Question 1: Fixed Income Valuation and Analysis / Fixed Income Portfolio Management (56 points)

a)
As regards the credit risk structure implied in the considered series of bonds, we can observe securities with a different "seniority":

- Bond A is a senior unsecured debt instrument. The redemption is not secured by any type of guarantee (this kind of bond is often called a "debenture"). The claim of the bond holders is over the assets of the issuer, after the creditors with legal priority, such as claims on assets, have been paid in full.

- Bond B is a senior guaranteed debt instrument. The payment of the obligations embedded in the bond is guaranteed by the assets of the issuer and by the assets of a third party (the guarantor). The quality of the guarantee depends on the creditworthiness both of the issuer and the guarantor.

- Bond C is a subordinated lower tier 2 debt instrument. The redemption will take place, in case of credit events, after the redemption of senior guaranteed and senior unsecured bonds.

Sorting the analyzed bonds according to an increasing credit risk, we will have:
1. Bond B, senior "guaranteed"
2. Bond A, senior unsecured
3. Bond C, subordinated Tier 2

b)
We can calculate the relative yield spread and yield ratio for the bonds A and C with respect to bond B as follows:

$$\text{Relative yield spread bond, A} = \frac{\text{Yield bond A - Yield bond B}}{\text{Yield bond B}} = \frac{2.75\% - 1.45\%}{1.45\%} = 0.8965$$

$$\text{Relative yield spread bond, C} = \frac{\text{Yield bond C - Yield bond B}}{\text{Yield bond B}} = \frac{4.00\% - 1.45\%}{1.45\%} = 1.7586$$

c)
Check for the static non-arbitrage condition: carry out the pricing of bond B using the base rates (swap spot curve) and the credit spread curve, comparing the present value of every cash flow to the market value of the zero coupon bonds with same credit risk and same maturities.

General formula is:

$$P = \sum_{j=1}^{n} \frac{CF}{\left(1 + R_{ZC}^{Tj} + \text{Spread}\right)^{Tj}} + \frac{100}{\left(1 + R_{ZC}^{Tn} + \text{Spread}\right)^{Tn}}$$

Pricing Bond B:

$$P = \frac{2}{(1+0.5\%+0.2\%)^1} + \frac{2}{(1+0.75\%+0.2\%)^2} + \frac{2}{(1+0.95\%+0.2\%)^3} + \frac{2}{(1+1.15\%+0.2\%)^4}$$
$$+ \frac{2+100}{(1+1.26\%+0.2\%)^5}$$
$$= 1.986 + 1.963 + 1.933 + 1.896 + 94.869 = 102.646$$

The value of the stripped zero coupon bond streams is
$$= 1.982 + 1.961 + 1.930 + 1.887 + 94.861 = 102.621$$

Hence, the static non-arbitrage condition is **not** verified.

In order to take profit of the static arbitrage it is necessary to sell bond B to buy on the market the full zero coupon strip of bank Y listed on the market and with equivalent credit risk:

- sell bond B at		+102.646
- buy zero coupon strip:		(-1.982)
		(-1.961)
		(-1.930)
		(-1.887)
		(-94.861)
	Total	(-102.621)

The price difference will be: (102.646 - 102.621) = 0.025 or 2.5 cents of price.

The net amount cashed for EUR 100 million nominal is equal to:
$$100\,\text{million} \cdot 0.025\% = \text{EUR}\,25{,}000$$

d)

d1)

The price of bond A is given by:

$$P = \sum_{j=1}^{n} \frac{CF}{(1+k)^{Tj}} + \frac{100\%}{(1+k)^{Tn}}$$
$$P_A = \frac{1.5}{(1+2.75\%)^{0.5}} + \frac{1.5}{(1+2.75\%)^1} + \frac{1.5}{(1+2.75\%)^{1.5}} + \frac{1.5}{(1+2.75\%)^2} + \frac{1.5}{(1+2.75\%)^{2.5}} + \frac{1.5+100}{(1+2.75\%)^3}$$
$$= 100.7688$$

The duration of Bond B, is given by:

$$\text{Duration} = D_{FW} = \sum_{t=1}^{T} \frac{PV(CF_t)}{P} \cdot t = \frac{1}{P} \cdot \sum_{t=1}^{T} \frac{t \cdot CF_t}{(1+k)^t}$$
$$D = \frac{1}{102.634} \cdot \sum_{t=1}^{5} \frac{(t) \cdot CF_t}{(1+1.45\%)^t}$$
$$= \frac{1}{102.634} \cdot \left(\frac{1 \cdot 2}{(1+1.45\%)^1} + \frac{2 \cdot 2}{(1+1.45\%)^2} + \frac{3 \cdot 2}{(1+1.45\%)^3} + \frac{4 \cdot 2}{(1+1.45\%)^4} + \frac{5 \cdot 102}{(1+1.45\%)^5} \right)$$
$$= 4.81$$

The convexity of bond C is given by:

$$\text{Convexity} = C^* = \frac{1}{P} \cdot \frac{1}{(1+k)^2} \cdot \sum_{t=1}^{T} \frac{(t) \cdot (t+1) \cdot CF_t}{(1+k)^t}$$

$$C^* = \frac{1}{75.992} \cdot \frac{1}{(1+4\%)^2} \cdot \frac{(7) \cdot (8) \cdot 100}{(1+4\%)^7} = 51.78$$

d2)

Relative price change (% price change) considering only the duration:

$$\frac{\Delta P}{P} = -\frac{D}{(1+k)} \cdot \Delta k$$

Using the calculated values:

$$\frac{\Delta P}{P} = -\frac{4.81}{(1+1.45\%)} \cdot 0.25\% = -1.185\%$$

[Using the given values:

$$\frac{\Delta P}{P} = -\frac{4.75}{(1+1.45\%)} \cdot 0.25\% = -1.171\%\,]$$

Relative price change (% price change) considering duration and convexity:

$$\frac{\Delta P}{P} = -D \cdot \frac{\Delta k}{1+k} + \frac{1}{2} \cdot C^* \cdot (\Delta k)^2$$

Using the calculated values:

$$\frac{\Delta P}{P} = -\frac{4.81 \cdot 0.25\%}{(1+1.45\%)} + \frac{1}{2} \cdot 27.679 \cdot (0.25\%)^2 = -1.177\%$$

[Using the given values:

$$\frac{\Delta P}{P} = -\frac{4.75 \cdot 0.25\%}{(1+1.45\%)} + \frac{1}{2} \cdot 27.679 \cdot (0.25\%)^2 = -1.162\%\,]$$

d3)

$$\text{Portfolio duration} = \sum_{i=1}^{n} w_i \cdot D_i$$

Using the calculated values:

$$\text{Portfolio duration} = \frac{1}{3} \cdot 2.89 + \frac{1}{3} \cdot 4.81 + \frac{1}{3} \cdot 7 = 4.9$$

[Using the given values:

$$\text{Portfolio duration} = \frac{1}{3} \cdot 2.7 + \frac{1}{3} \cdot 4.75 + \frac{1}{3} \cdot 7 = 4.82\,]$$

$$\text{Portfolio convexity} = \sum_{i=1}^{n} w_i \cdot C_i$$

Using the calculated values:

$$\text{Portfolio convexity} = \frac{1}{3} \cdot 9.47 + \frac{1}{3} \cdot 27.679 + \frac{1}{3} \cdot 51.775 = 29.641$$

[Using the given values:

$$\text{Portfolio convexity} = \frac{1}{3} \cdot 9.5 + \frac{1}{3} \cdot 30 + \frac{1}{3} \cdot 50 = 29.833]$$

d4)

If credit spreads rise, the price of the bonds sink. Therefore bond D is the best choice, having a medium-long term horizon and offering a protection against a fall in the prices. Moreover, being long a put option, if volatilities rise the long position will profit from an increase in the option prices.

Comparing bond D and C they have the same yield to maturity (4%), but bond D has the put option embedded (higher total value compared to bond C).

Bond E gives the option to reimburse the bond in one year time to the issuer at 1% yield. This bond is not the correct choice because if credit spreads rise, the price of the bonds sinks, but bond E does not offer a protection against a fall in the prices.

Bond F would allow the investor to protect the invested capital thanks to the shortest duration, but it doesn’t respect the constraint to implement a medium/long term investment.

e)

e1)

Forecasting a YTM curve flattening between the 2yrs and 10yrs bonds, the duration trade involves:

- selling Bond G 4.00% 2/2017 (2yrs)
- buying Bond K 5.00% 3/2025 (10yrs)

Considering an amount of nominal EUR 10,000,000 for the 2yrs bond at inception, the strategy will be the following:

$$\text{Nom. amount Bond 2yr} \cdot \text{Price} \cdot \text{Mod.duration} = \text{Nom. amount Bond 10yr} \cdot \text{Price} \cdot \text{Mod.duration}$$

$$10{,}000{,}000 \cdot 108.7 \cdot 2.33 = X \cdot 122.1 \cdot 8.17$$

$$X = 2{,}539{,}000$$

=> Nominal amount to buy of Bond 5% 3/2025

Countervalue of the bond G 2yrs to sell: $\frac{10{,}000{,}000 \cdot 108.7}{100} = \text{EUR } 10{,}870{,}000$

Countervalue of the bond K 10yrs to buy: $\frac{2{,}539{,}000 \cdot 122.1}{100} = \text{EUR } -3{,}100{,}119$

e2)
In order to implement a duration weighted "bullet to barbell" strategy, the trades involved are the following:

- Sell Bond I 4.50% 2/2020 (5yrs)
- Buy Bond G 4.00% 2/2017 (2yrs)
- Buy Bond K 5.00% 3/2025 (10yrs)

At inception we go short EUR 11.45 million of the 5 year bond, and we go long X million of the 2 yearbond and Y million of the 10 year bond. We have:

$$\begin{cases} 11.45 \cdot 4.85 = \mathrm{X} \cdot 2.33 + \mathrm{Y} \cdot 8.17 & \text{(zero portfolio duration)} \\ 11.45 = \mathrm{X} + \mathrm{Y} & \text{(zero portfolio value)} \end{cases}$$

$$\Rightarrow \begin{cases} 11.45 \cdot 4.85 = \mathrm{X} \cdot 2.33 + (11.45 - \mathrm{X}) \cdot 8.17 \\ 11.45 - \mathrm{X} = \mathrm{Y} \end{cases} \Rightarrow 11.45 \cdot (8.17 - 4.85) = X \cdot (8.17 - 2.33)$$

$$\Rightarrow X = 11.45 \cdot \frac{(8.17 - 4.85)}{(8.17 - 2.33)} = 6.509 \qquad \mathrm{Y} = 4.941$$

The nominal amount of bond 2yrs to buy: $\frac{6{,}509{,}000}{1.087} = \text{EUR } 5{,}988{,}000$

The nominal amount of bond 10yrs to buy: $\frac{4{,}941{,}000}{1.221} = \text{EUR } 4{,}047{,}000$

Question 2: Derivative Valuation and Analysis (39 points)

[Note: the option prices have been obtained by B&S using $S_0 = 100$, $r = 1\%$, $t = 0.5$, $\sigma = 25\%$, $y = 0$]

a)

a1)

The initial cost of the option strategy is:

$$100 \cdot (-13.15 + 9.93 + 5.18 - 3.59) = -163\,\text{CHF}$$

a2)

The value V_T of the strategy at maturity is:

$$V_T = 100 \cdot [\max(S_T - 90;0) - \max(S_T - 95;0) - \max(S_T - 105;0) + \max(S_T - 110;0)]$$

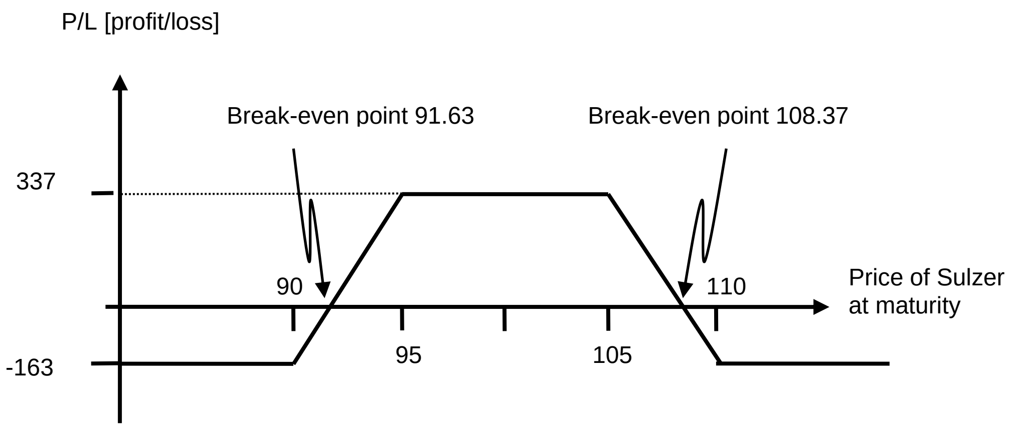
$$= \begin{cases} 0 & (\text{if } S_T < 90) \\ 100 \cdot (S_T - 90) & (\text{if } 90 \le S_T < 95) \\ 500 & (\text{if } 95 \le S_T < 105) \\ 100 \cdot (110 - S_T) & (\text{if } 105 \le S_T < 110) \\ 0 & (\text{if } S_T \ge 110) \end{cases}$$

a3)

$$\text{maximum profit} = \text{maximum value} - \text{initial cost} = 500 - 163 = 337$$

$$\text{maximum loss} = \text{initial cost} = -163$$

break-even points = 91.63 and 108.37



[Note: In the current exercise we are neglecting the interests on option premiums in order to simplify the calculations and understandings of the results.]

b)

The buyer of the condor believes in a stationary underlying, which at maturity lies between 95 and 105, or at least in the interval [91.63, 108.37].

The condor has a limited risk [maximum loss CHF 163] and consists in a non-directional strategy which realizes a profit in case the underlying remains stationary.

c)
Condor =
long bull call spread $[K_1 = 90, K_2 = 95]$ + short bull call spread $[K_3 = 105, K_4 = 110]$.

d)
Buy put with strike $K_1 = 90$, sell put with strike $K_2 = 95$, sell put with strike $K_3 = 105$ and buy put with strike $K_4 = 110$.

e)
$P = C - S + Ke^{-r \cdot t}$, therefore:

$$\begin{aligned} P_{90} - P_{95} - P_{105} + P_{110} &= C_{90} - C_{95} - C_{105} + C_{110} + (90 - 95 - 105 + 110) \cdot e^{-r \cdot t} \\ &= C_{90} - C_{95} - C_{105} + C_{110} \end{aligned}$$

i.e. they have the same initial cost.

f)
The delta of the strategy is: $\Delta = 100 \cdot [0.76 - 0.66 - 0.44 + 0.34)] = 0$.
Therefore the strategy is already delta neutral and there is no need to take any position on the underlying security.

Question 3: Derivatives/Derivatives in Portfolio Management (33 points)

a)

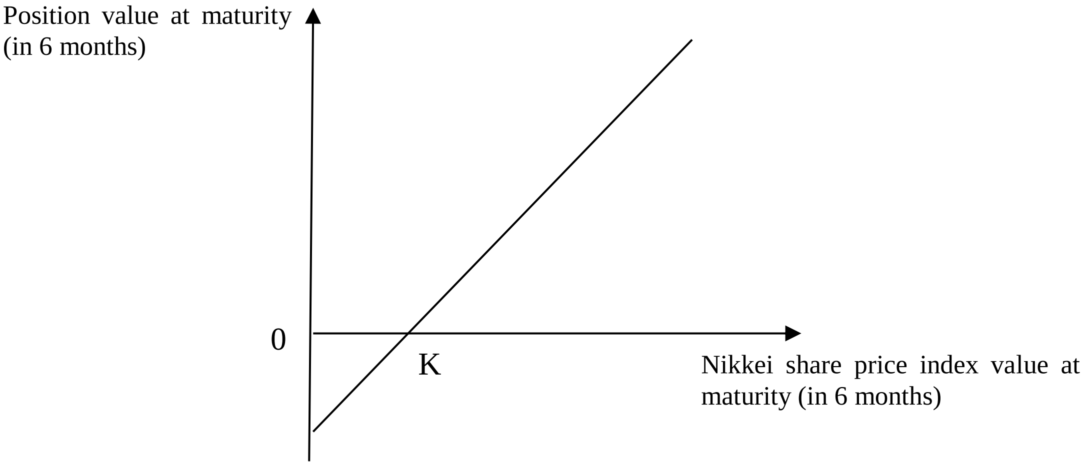
a1)

Dividends and transaction costs can be ignored, so the no-arbitrage price of the Nikkei average futures F_0 maturing in 6 months is given as follows:

$$F_0 = S_0 \cdot \left(1 + \frac{r_f}{2}\right) = 20000 \cdot \left(1 + \frac{4\%}{2}\right) = 20400 \text{ JPY}$$

a2)

The position created by selling 1 trading unit of the put option and buying 1 trading unit of the call option results in the following maturity payoff diagram.



a3)

Under the no-arbitrage condition, the following relationship holds true for the price of the European put option with a maturity of 6 months (0.5 years) and a strike price K, the price of the call option with the same strike price and maturity, the futures price and interest rates r_f (annualized):

$$C_0(K) - P_0(K) = \frac{F_0 - K}{1 + \frac{r_f}{2}} = \left(S_0 - \frac{K}{1 + \frac{r_f}{2}}\right)$$

b)

b1)

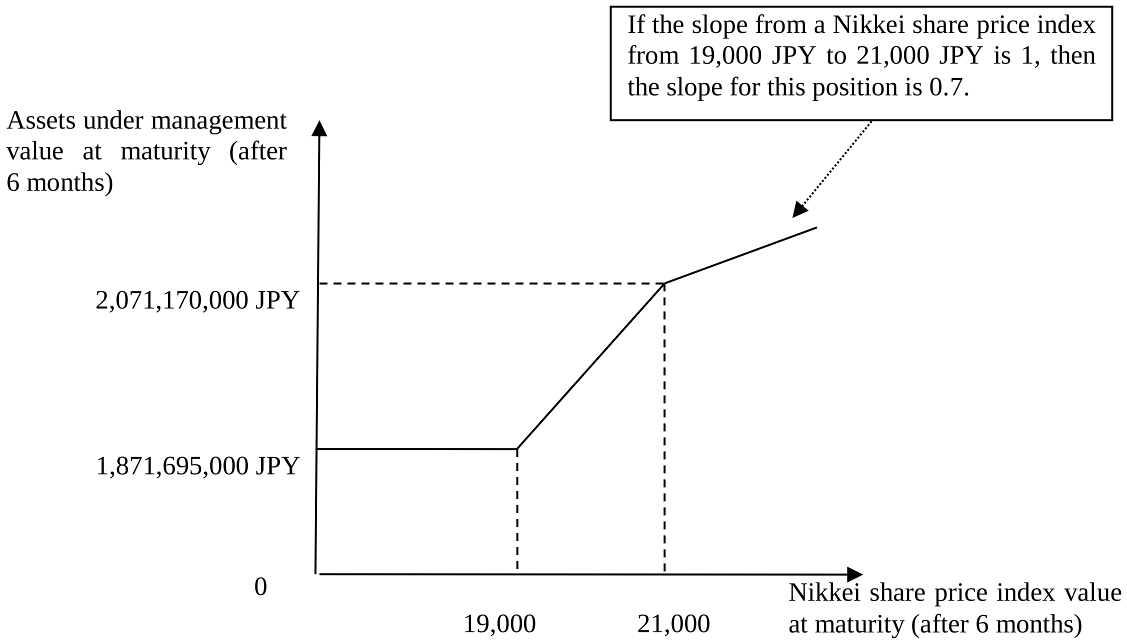
$$N_P = \frac{\text{Portfolio value}}{\text{Index level} \cdot \text{Option contract size}} = \frac{2 \text{ billion}}{20{,}000 \cdot 1000} = 100$$

Mr. A should buy 100 trading units of put option.

The put option purchasing cost is 540 yen per-unit put option, and factoring in interest rates, the value in 6 months is $540 \cdot (1+2\%) = 550.8$ JPY. The value found by subtracting the purchasing cost from the strike price of the put option is the achievable floor per-unit option, which is $19{,}000 - 550.8 = 18{,}449.2$ JPY. Therefore, the achievable floor for the portfolio net position is 1,844.92 million JPY (100,000 times the unit value).

b2)
Selling 30 trading units (= 30,000) of a call option with a strike price of 21,000 JPY will produce proceeds from the sale of $875 \cdot 30{,}000 = 26.25$ million JPY. The value after investment at the risk-free rate for 6 months is $26.25 \cdot (1+2\%) = 26{,}775{,}000\,\text{JPY}$, and this is added to the floor found in b1), resulting in an achievable floor of:
$1{,}844{,}920{,}000 + 26{,}775{,}000 = 1{,}871{,}695{,}000\,\text{JPY}$

The payoff diagram is shown below.



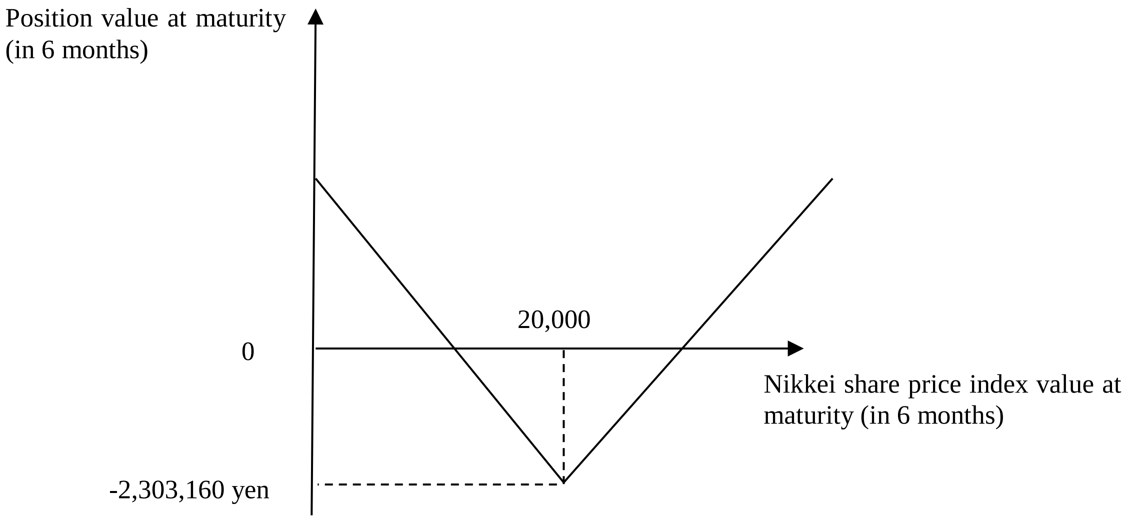
b3)
To use a futures dynamic hedge to synthesize the put option position proposed in b1), (delta of a put option with strike price of 19,000 JPY/delta of the future) units of the future are sold per unit put option. They are sold because the put option delta is negative.

Number of futures per unit put option = $\dfrac{-0.28}{0.98} = -0.286$

c)

c1)

Below is the relationship between return on the position and the equity price index in 6 months.



In this case, the minimum return is equivalent to the option purchasing cost plus the interest rate paid under the risk-free rate during the 6 months:

$$-[C_0(K)+P_0(K)]\cdot\left[1+\frac{r_f}{2}\right]=-(1325+933)\cdot 1000\cdot 1.02=-2{,}303{,}160\,\text{JPY}$$

c2)

The return on the position in c1) will be positive and increase in direct proportion to the change (increase of decrease) in the Nikkei share price index in 6 months and hence the larger the variance from the current price of 20,000 JPY, the higher will be the payoff. Therefore, Mr B is expecting higher volatility than the implied volatility calculated from the option premiums on the market.

Question 4: Portfolio Management **(40 points)**

a)

$$\frac{\overline{R}_A - R_F}{\sigma_A} = \frac{0.12 - 0.01}{\sqrt{0.09}} = 0.3667$$

$$\frac{\overline{R}_B - R_F}{\sigma_B} = \frac{0.06 - 0.01}{\sqrt{0.0625}} = 0.2$$

$$\frac{\overline{R}_C - R_F}{\sigma_C} = \frac{0.085 - 0.01}{\sqrt{0.04}} = 0.375$$

The average return earned in *excess* of the risk-free rate per unit of volatility or total risk, known as the Sharpe Ratio, is one of the most used performance measure and can be used to do a first analysis of more risky alternatives with different levels of risk and return that are not directly comparable. The greater the value of the Sharpe ratio, the more attractive the single investment.

This ratio provides a measure of **absolute** risk, not suitable for the evaluation of financial assets directly linked to a benchmark (and of course for well diversified portfolios). In this case it is appropriate to apply measures of relative risk as the TEV or the IR.

In our example, the component C is the most attractive alternative.

b)

Let us compute the expected return and the volatility of the two portfolios P1 and P2:

$$\overline{R}_{P1} = x_A \cdot \overline{R}_A + x_B \cdot \overline{R}_B + x_C \cdot \overline{R}_C = 0.3 \cdot 0.12 + 0.5 \cdot 0.06 + 0.2 \cdot 0.085 = 0.083$$

$$\overline{R}_{P2} = \frac{1}{3} \cdot \left(\overline{R}_A + \overline{R}_B + \overline{R}_C\right) = \frac{1}{3} \cdot 0.265 = 0.0883$$

$$\sigma_{P1} = \left(\sum_{i,j(P1)} x_i \cdot x_j \cdot \sigma_{ij}\right)^{0.5} = \begin{pmatrix} 0.3^2 \cdot 0.09 + 0.5^2 \cdot 0.0625 + 0.2^2 \cdot 0.04 + 2 \cdot 0.3 \cdot 0.5 \cdot 0.0175 \\ + 2 \cdot 0.3 \cdot 0.2 \cdot 0.041 + 2 \cdot 0.5 \cdot 0.2 \cdot 0.0225 \end{pmatrix}^{0.5} = 0.2$$

$$\sigma_{P2} = \left(\sum_{i,j(P2)} x_i \cdot x_j \cdot \sigma_{ij}\right)^{0.5} = \frac{1}{3} \cdot \left(0.09 + 0.0625 + 0.04 + 2 \cdot 0.0175 + 2 \cdot 0.041 + 2 \cdot 0.0225\right)^{0.5}$$
$$= 0.1985$$

According to the risk/return tradeoff, it is preferable to invest in portfolio P2 as it strictly dominates the portfolio P1. In fact, $\overline{R}_{P2} > \overline{R}_{P1}$ and $\sigma_{P2} < \sigma_{P1}$.

c)

To evaluate the diversification effect for each combination of two stocks (in terms of variance) we have to compare the ratio of the two stock volatilities with the corresponding correlation coefficient. If the following relation is verified, then we have diversification effect:

$\rho_{12} < \frac{\sigma_1}{\sigma_2}, \quad \sigma_2 > \sigma_1.$

i.e. we can have $\sigma_P < \sigma_1 < \sigma_2$

[Note: this result comes from the analytical solution of the Markovitz minimum variance portfolio in the case of two risky alternatives. In particular, we solve the inequality $x_1^* < 1$, where $\left(x_1^*, x_2^*\right)$ are the solutions of the minimum variance problem with the balance constraint $x_1^* + x_2^* = 1$.]

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \cdot \sigma_B} = \frac{0.0175}{\sqrt{0.09} \cdot \sqrt{0.0625}} = 0.2333 < \frac{\sigma_B}{\sigma_A} = 0.8333$$

$$\rho_{AC} = \frac{\sigma_{AC}}{\sigma_A \cdot \sigma_C} = \frac{0.041}{\sqrt{0.09} \cdot \sqrt{0.04}} = 0.6833 > \frac{\sigma_C}{\sigma_A} = 0.6667$$

$$\rho_{BC} = \frac{\sigma_{BC}}{\sigma_B \cdot \sigma_C} = \frac{0.0225}{\sqrt{0.0625} \cdot \sqrt{0.04}} = 0.45 < \frac{\sigma_C}{\sigma_B} = 0.8$$

Therefore, the combinations (A,B) and (B,C) allows to benefit from the diversification effect, while the combination of (A,C) does not.

d)

The single-index model expected return on portfolios P is defined as follows:

$$\overline{R}_P = \sum_i x_i \cdot \overline{R}_i = \sum_i x_i \cdot \left(\alpha_i + \beta_i \cdot \alpha_M\right) = \sum_i x_i \cdot \alpha_i + \alpha_M \cdot \sum_i x_i \cdot \beta_i = \sum_i x_i \cdot \alpha_i + \alpha_M \cdot \beta_P$$

Then, we have:

$$\beta_P = \sum_i x_i \cdot \beta_i$$

$$\beta_{P1} = 0.3 \cdot 0.76 + 0.5 \cdot 1.3 + 0.2 \cdot 0.97 = 1.072$$

$$\beta_{P2} = \frac{1}{3} \cdot \left(0.76 + 1.3 + 0.97\right) = 1.01$$

$$\overline{R}_{P1} = \sum_{i\,(P1)} x_i \cdot \alpha_i + \alpha_M \cdot \beta_{P1} = \left(-0.3 \cdot 0.025 - 0.5 \cdot 0.014 + 0.2 \cdot 0.03\right) + 0.055 \cdot 1.072 = 0.05046$$

$$\overline{R}_{P2} = \sum_{i\,(P2)} x_i \cdot \alpha_i + \alpha_M \cdot \beta_{P2} = \frac{1}{3} \cdot \left(-0.025 - 0.014 + 0.03\right) + 0.055 \cdot 1.01 = 0.05255$$

When we compare the theoretical expected return on the two portfolios, we can state that P2 is more attractive than P1. In this case we cannot analyze the risk/return tradeoff as in the above point b). This is due to the different risk measures applied in the two models. Risk-adjusted return, or Sharpe's ratio, is an absolute performance measure, while the beta represents a relative risk measure. In particular, the parameter beta measures the systematic risk of the portfolio with the market (or benchmark), that is to say the average change in the portfolio return as a result of changes in the performance of the market.

In the present example, the beta indicates that both portfolios are *aggressive* with respect to the EUROSTOXX 50 $(\beta_P > 1)$, but P1 has a risk profile more aggressive than P2.

e)

The target is to calculate the portfolio variance for each portfolio composed of only two risky assets. Given that $\sigma_i^2 = \beta_i^2 \cdot \sigma_M^2 + \sigma_{\varepsilon_i}^2$, we obtain the following formula:

$$\sigma_P^2 = \sum_i x_i^2 \cdot \beta_i^2 \cdot \sigma_M^2 + \sum_i x_i^2 \cdot \sigma_{\varepsilon_i}^2 = \beta_P^2 \cdot \sigma_M^2 + \sum_i x_i^2 \cdot Q_i$$

Now, we have to compute the portfolio weights that define a neutral portfolio with respect to the benchmark EUROSTOXX 50, that is to say $\beta_P = 1$. From the formula to calculate the beta of the portfolio we obtain:

$$x_A \cdot \beta_A + (1 - x_A) \cdot \beta_B = 1 \Leftrightarrow x_A \cdot (\beta_A - \beta_B) = 1 - \beta_B \Leftrightarrow x_A = \frac{1 - \beta_B}{\beta_A - \beta_B}$$

$$\Rightarrow x_A = \frac{1 - \beta_B}{\beta_A - \beta_B} = 0.5556; \qquad x_B = 1 - x_A = 0.4444$$

$$\sigma_{(AB)}^2 = 1^2 \cdot 0.12 + 0.5556^2 \cdot 0.15 + 0.4444^2 \cdot 0.07 = 0.1801$$

Question 5: Portfolio Management (12 points)

a)
While it is correct that “Each was given USD 100 million to start, each was given another USD 50 million, and each had USD 50 million taken away,” the timing of the cash flows differed. Both the total return over the three years, and the suggested internal rate of return criterion are affected by the timing of the cash flows – which were not under the control of the managers and should not be part of the performance review.

b)
The review should be based on the time weighted return (TWR), as computed below:

Year 1	Portfolio A	Portfolio B
Value at start, year 1	100,000,000	100,000,000
Value end of year 1	112,000,000	106,000,000
$1 + TWR_1$	1.12	1.06
Year 2		
Value at start, year 2	162,000,000	56,000,000
Value end of year 2	178,200,000	62,720,000
$1 + TWR_2$	1.10	1.12
Year 3		
Value at start, year 3	128,200,000	112,720,000
Value end of year 3	134,610,000	123,992,000
$1 + TWR_3$	1.05	1.10

The total annualised TWR’s are:
Portfolio A = $\sqrt[3]{1.12 \cdot 1.10 \cdot 1.05} - 1 = 8.96\%$

Portfolio B = $\sqrt[3]{1.06 \cdot 1.12 \cdot 1.10} - 1 = 9.30\%$

Hence, the portfolio manager B had the better performance over the three years.